Math 1A
Midterm 2 Review

You should be able to find any derivative from this chapter.

3.1	3-32
3.2	3-34
3.3	1-16
3.4	1-54
3.5	5-20, 25-32, 49-62
3.6	2-30, 39-52
3.11	30-45
3.REV	1-50

For section 3.9 (Related Rates), the review material is all the assigned homework plus the examples we have been working on in lecture.

Knowing how to find derivatives is not enough, because once again, there will be very few questions which simply ask you to find a derivative. You should also be able to solve all the following types of problems.

[1] Estimate $\csc 0.5$ using a linear approximation chosen at an appropriate point.

[2] If $y = \dfrac{1}{x^2}$, find dx, Δy and dy if $x = 2$ and $\Delta x = 0.5$.

[3] Find $\dfrac{d^3}{dx^3} \sec x$. **Simplify your answer.**

[4] The position of an object at time t is given by the function $s(t) = \dfrac{2t^3 + 4t^2 - 3}{\sqrt{t}}$ for $t \geq 0.5$.

[a] Find the velocity of the object at time $t = 1$.

[b] Find the acceleration function. **Simplify and factor your answer.**

[5] Find the equations of the tangent lines to the curve $y = 1 + x^3$ that are perpendicular to $x + 12y = 1$.

[6] The line $y = 3x - 4$ is tangent to a quadratic function at the point $(1, -1)$. Find the equation of the tangent line to the quadratic function at $(2, 4)$.

[7] If $f(x) = \dfrac{x^3}{1 + x^2}$, find $f''(1)$.

[8] The following table gives values and derivatives of two functions at various inputs.

x	–3	–2	–1	0	1	2	3	4
$f(x)$	–2	0	2	4	–3	–1	1	3
$f'(x)$	4	–1	–3	2	–4	3	–2	1
$g(x)$	–1	1	3	–3	4	–2	0	2
$g'(x)$	2	4	–4	–1	3	1	–3	–2

[a] If $k(x) = x^3 f(x)$, find the equation of the tangent line to $y = k(x)$ at $x = 2$.

[b] If $j(x) = \dfrac{x^2}{f(x)}$, find the equation of the tangent line to $y = j(x)$ at $x = -1$.

[c] If $m(x) = \tan^{-1}(g(x))$, find the equation of the tangent line to $y = m(x)$ at $x = -3$.

[d] If $n(x) = g(f(x))$, find the equation of the tangent line to $y = n(x)$ at $x = 4$.

[9] If $h(x)=f(x)g(x)$, find formulae for $h''(x)$ and $h'''(x)$. Based on your answers, guess a formula for $h^{(4)}(x)$ (the fourth derivative of $h(x)$.

[10] Find all x-coordinates in the interval $[0, 2\pi]$ where the tangent line to $f(x)=4x-3\tan x$ is horizontal.

[11] If $f(x)=xg(x^2)$, find a formula for $f''(x)$. Your answer may involve g, g' and/or g''.

[12] Find the equation of the tangent line to $(1+x^2y^3)^5=x^4e^y$ at $(-1, 0)$.

[13] Show that $y=ax^4$ and $x^2+4y^2=b$ are orthogonal trajectories. **See section 3.5, questions 65-68.**

[14] If $y=(\sin x)^{\frac{1}{x}}$, find $\dfrac{dy}{dx}$.

[15] The limit $\lim\limits_{h\to 0}\dfrac{(h-1)e^{1-h}+e}{h}$ is the derivative of some function $f(x)$ at some point $x=a$. Find the function, the point, and the value of the limit, by evaluating the corresponding derivative.

You must also know the following proofs.

Proofs

quotient rule
- using the definition of the derivative

derivatives of $\sin x$, $\cos x$, $\tan x$, $\csc x$, $\sec x$ and $\cot x$
- using the definition of the derivative, without using the derivatives of any other trigonometric function
- you may use the limits $\lim\limits_{h\to 0}\dfrac{\sin h}{h}=1$ and $\lim\limits_{h\to 0}\dfrac{\cos h-1}{h}=0$ without proving them

derivatives of $\tan x$, $\csc x$, $\sec x$ and $\cot x$
- using the quotient rule
- you may use the derivatives of $\sin x$ and $\cos x$ without proving them

derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, and $\ln x$
- using implicit differentiation
- you may use the derivatives of $\sin x$, $\cos x$, $\tan x$ and e^x without proving them